UNIVERSITY OF CAMBRIDGE

Cambridge University History of Mathematics Society

presents an illegal, immoral, *and* fattening lecture series,
without the endorsement, approval, or approbation of University authorities on:

A HISTORY OF MATHEMATICAL IDEAS
FROM
CLASSICAL ANTIQUITY TO THE ENLIGHTENMENT

To be held on

Wednesdays and Fridays, Michaelmas & Lent Terms 2024-25

Well, to the First Years, welcome to Cambridge, and to the Second and Third years, welcome *back* to Cambridge, and to anyone else reading this or watching these lectures online, welcome to Cambridge education, sitting on your couch watching videos. Since these lectures are on-line, I am happy to welcome people online who are not Cambridge undergraduates, although I should warn you that the tone and tenor of the lectures is directed to a local audience. It's lovely to have you all back up in Cambridge, although I can't tell if the people on Zoom are in Cambridge or Seoul, or anywhere between … so "coming [back] up to Cambridge" is something of a misnomer. But, with thanks to the Cambridge University History of Maths Society, who now organise my history of maths and history of science courses, welcome to Cambridge In-person and On-line, Inc., purveyor of education to the interested elites everywhere.

This year we will continue as normal – as if anything about Cambridge could be described as normal – with these lectures on-line; *to begin with there will be no face-to-face lectures*. The History of Maths Society (HoMSoc, obvs) is trying to find us a lecture hall (at no cost) somewhere in the University where we can go hybrid and have face to face lectures, streamed on Zoom. That won't happen for a couple of weeks at least, so we will start on-line, and go hybrid whenever we can.

Links to the lectures on-line and other news and announcements will be in my irregular emails to those that attend these lectures; you need to sign up to the email list (QR Code below). As things stand, I shall not put these lectures up on YouTube – the task of editing them into something tolerably adequate for YT is more than I'm willing to do – although if any of you are adept at video editing and would like to help, please let me know: I might reconsider. Lectures are recorded, however (on the occasions when I don't screw something up) so if you miss a lecture and want to catch up you will be able to get the recording. Location of recorded lectures will be given in the emails you will get if you sign up to them. If you don't sign up to the email list, you will be cast into the Outer Darkness, and not know what is going on.

To those of you new to Cambridge, life returns to normal at the beginning of Michaelmas Term: this includes the glamorous social life we all have here, endless power-networking, organising World Revolution (or your future career as a boring banker or Hedge Fund billionaire), sex'n'drugs'n'rock'n'roll (that you should be so lucky), fancy dress dinner parties in bib'n'tucker,

erudite (and/or pretentious) seminars and talks about everything, and cycling in the bitterly cold rain of Cambridge. Sobriety optional. If you're new here, you will soon learn what I mean.

And for those who have been here, you also know that loneliness, depression, Impostor Syndrome, fatigue, alienation, vitamin deficiencies, despair, total immersion in maths and even some very occasional moments of actually understanding something in lectures are also included, free of charge. And you're a mathmo, so you aren't going to get much of the partying and glamour of all those Hooray Henrys and Henriettas anyway, so the glamour of your social life is … well, relative. Welcome to Cambridge.

Lecture topics this year

As the years go by and slowly some glimmer of wisdom has managed to seep through to me, I have learned not to promise that certain lectures will cover particular topics, as the inevitable failure to keep precisely (or even vaguely) to the announced programme and any sort of timetable makes *me* feel guilty and gives rise to an appreciation on the part of students that the University is completely disorganised and held together by string and sealing wax. Since the entire Cambridge system is carefully designed to hide this phenomenon, it is considered unprofessional to make it too obvious to students. You are supposed to discover this on your own, slowly and painfully. Hence, I announce here some of the topics that I will cover in the lectures that follow*[1], but this list is neither complete nor shall it be deemed anything so firm as an actual *promise* or contract to talk about these topics.

1 For ***introductory stu*** , I will give an overview of the history that I intend to cover over these lectures, possibly discuss a couple of questions about the foundations or nature of mathematics (in a very abbreviated way), and briefly some points about epistemology and proof and mathematics; I will, obviously, avoid trying to explain definitively the underlying nature or ontology of mathematics. I will also at some point want to talk about Whig historiography (or just give you a 600-page handout that you won't read) and why we don't do that sort of thing – the Whig stuff, I mean – because it is both *rude* and *bad*, and causes carcinogenic halitosis; and also is intellectually crass.

Then I might mention extremely briefly the early Chinese development of mathematics (which is awesome and deserves its own lecture course), and then move on to talk for a lecture or two about two (of the many) early 'practical' mathematical traditions: the ancient Egyptians and Mesopotamians from about 4,000BC to maybe 500BC.

2 This will take a week or so, and then I will talk about the Greeks in some detail – primarily because I think their maths is cool and I have thought a lot about it. I will cover topics such as:

(i) ***Greek mathematics generally*** (sources, transmission, and survival phenomena)
(ii) rethinking [read: *burying*] the story of the Pythagoreans;
(iii) an alternative view of early geometry and its status;
(iv) Parmenides and reasoned sceptical attack;
(v) Hippocrates of Chios, not Cos);
(vi) ~~Hippocrates of Chios, not Cos~~);
(vii) the development of the concept of proof;
(viii) three classical problems (circle squaring, cube doubling, angle trisecting);

*[1] although absolutely no commitment to precision or exactitude is made, and the lecturer will not be held responsible for deviations, additions and omissions from this programme; this does not affect your non-existent statutory rights

(ix) Zeno and Democritus on infinite division;
(x) the problem (it's not a crisis) of proportion.

Then I will talk about the profound problems that emerge from the Greek notion of space, and in particular the Greek worries about the nature of space because of the problem of *curvilinear quadrature* (how can you relate curvilinear space to rectilinear space?); this turns out to be one of the most important problems of all mathematics, and will remain problematic until the late 19th century. Thus:

(i) squaring the circle; Hippocrates, Antiphon & Bryson;
(ii) Eudoxus' method of exhaustion;
(iii) How Eudoxus' method of exhaustion might have developed, Archimedes' method of exhaustion;
(iv) the problem of heuristics and method: Archimedes' *Mechanical Method* [which was lost, not lost, lost, found, lost, not lost]

In all of this there is the continuing problem or story of the development of proof, and eventually axiomatics … and *why* anyone would do something so odd as axiomatize geometry. The question of ***why*** Euclid did Euclid is not considered by historians of mathematics, but if I have enough chocolate, *you* are going to find out.

Finally, I will give a very cursory glance at ***later Greek mathematics*** (Apollonius, Diophantus and Pappus, for example; commentators, creative mathematicians, and then the incredibly important, world-changing story of the survival and transmission of texts). I will try to compress this part of the story into less than 24 lectures.

3 Then I shall look very briefly at early Muslim mathematics and the general problem of the transmission of ancient Greek texts; transmission directions (although it is much more trendy to talk about transmission vectors) and their problems, i.e.

(i) the sources and context of Muslim-Arab mathematics and the discontinuity of research traditions:
(ii) translators, arithmetic and algebras; al-Khwarizmi and the Algebra,
(iii) algebra & geometry confused; and possibly some
(iv) applications of mathematics.

I will also discuss the *General Theorem*: medieval Muslim scientific culture was seriously cool. This theorem is discussed more fully in the History of Science for Mathmos course that I will give on *Science in Early Islam* on Thursdays in a couple of years. You missed it last year.

4 Next I shall look at (western) ***Medieval, Renaissance, neo-classical & Baroque [pre-calculus] maths***, which consists of an examination of the life and times and development of some more *discontinuous research traditions* (you might notice that DRTs are an important thing to watch for in the history of maths):

(i) **Medieval and early Renaissance mathematics**; the complex transmission vectors of texts and the medieval project in mathematics: (1) the "high maths" medieval tradition and uses of geometry,

(ii) and (2) the traditions of practical mathematics and teaching arithmetic; the subsequent cossist and abbacist "low maths" traditions and the equation solving arts, and
(iii) then all of this gets seriously important after the general solution to the cubic and the development of decent algebraic notation,
(iv) the new commercial enterprise of printed 'algebra' or cossist texts
(v) the changing status of 'algebra' and the science of equations, and its consequences for mathematics and the rest of the cosmos.

5 Then I will talk about a further transmission vector of ancient Greek mathematics into Renaissance Europe in the first half of the 15th century, and more generally about ***Renaissance humanism and mathematics***, which takes us once again to another discontinuous research tradition and the new mathematics of curves (that is, *Son of Archimedes* or Method of Exhaustion II):

(i) the late Renaissance re-construction of the ancient geometrical research tradition,
(ii) assorted corruptions and the search for the hidden heuristic;
(iii) the changing scope and status of mathematics;
(iv) the subtle but extraordinary effects of the slow but steady seepage of algebra to the brain and a faith in formal manipulation; and finally,
(v) the scandal of the missing heuristic.

Once we find ourselves slipping quietly into the Seventeenth century, I will continue this story of the lost heuristic, and geometry and algebra talking to each other with its consequences:

(i) Discontinuous research traditions and infinitesimals: Kepler, Cavalieri and others, and a radical new heuristic ... or, more generally, substance abuse in the 17th Century.
(ii) Then Fermat & Descartes (and what *was* he smoking?), safe sex and safer alternatives to the geometry of infinitesimals: what I call the tradition of '**equation bashing**' – a small but dangerous ambition to study curves using algebraic geometry;
(iii) an emerging mid-century consensus and lack of consensus on the theory of curves. Here you will find out what a GAGTOC is, which is something that will change your life, followed by some mildly polemical remarks about:
(iv) what the calculus is and is not, and how *not* to view its history. Note: on historical maps this area is labelled: "*Dragons and Bad Historians Found Here*"

6 Further on the topic of what the calculus is not, I shall talk about ***the calculus & early Enlightenment stu*** , which will be a very rapid tour through such topics as :

(i) Newton (a typical Trinity mathmo) and
(ii) Leibniz (ugh: another *foreigner*): both studied general algebraic-geometric theories of properties of curves;
(iii) followed by the story of the early calculus - Malebranch, l'Hôpital & the Basle group;
(iv) early foundational problems;
(v) Fontenelle and more early signs of formalist thinking;
(vi) early limitations to the new algebraic geometry of curves

(vii) and generally will try to subtly suggest that neither Newton nor Leibniz discovered what we think of as the calculus. But that's just me, trying to be awkward and annoying as usual.

7 As if that were not bad enough, I will talk briefly about ***mathematics and the Enlightenment*** (roughly 1700 to 1800), which is to say:

(i) a compare and contrast of the life and times of higher mathematics and mathematical physics in France and England: institutions, education, patronage, and the new language of mathematics (the weird, incredibly important story of analytic *vs.* synthetic intuitions);

(ii) the foundations of the calculus in the middle and later 18th century, reception of the calculus and then Euler's programme, analysis as a way of life, and 'ontological agnosticism'; and then maybe something on

(iii) analytical mathematical physics – from 17th century mathematical mechanics via Newton to Euler and d'Alembert's ambitions, to Lagrange's analytical mechanics and Laplace's universal analytical point-force mechanics.

and, more generally, I will talk about the new imperialism of mathematics and 'higher mathematics', and the unspoken Enlightenment project to mathematise more and more areas of science. The spread of irreligion (not atheism!). The sciences of chemistry, human society, animal behaviour, even politics get caught up in these ambitions, let alone Laplace and the dream of determinism. It was sort of absurd, the fantasies of tech bro's thinking they could solve everything. Sounds familiar. And at the same time, there was a general sense of depression: for mathematics, "the mine had gone to deep".

How much I talk about Enlightenment pure and applied mathematics, foundations, and the growing imperialist hegemony of mathematics in the sciences will depend on how quickly you listen: if you listen slowly, we will fall behind. Please note how this makes it *your* fault not *mine* if we don't finish the course by the end of Lent. We call this 'guilt transference' in the trade.

8 In past years I have not really lectured much about the 19th century, but occasionally I have given a few rather general lectures about maths in the 19th century and the way that mathematics changed completely and totally in the middle decades of the century. We can see if there is much or any appetite for any, or some, or more lectures on the 19th century, and if any students would like to contribute some presentations on interesting and comprehensible topics. I make no promises on this, and we can talk about this further in Lent Term.

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If all of that sounds a bit heavy, don't be fooled. This was once a Cambridge University lecture course, so it isn't exactly supposed to be trivial or bog-standard, and is designed to go beyond what you will find in the secondary literature. If you want to know what is in the standard secondary literature, there are libraries for that. We generally presume that you can cover that stuff on your own if you are interested. Or ignore it because so much of it is poor quality.

~~However, this is a survey of the~~ history of *mathematical ideas and the culture of mathematics*, of the intellectual context and styles of mathematical work, ***not*** (let me repeat that in case you didn't get it: ***NOT***) a course in mathematical technicalities. This may disappoint some mathmos (who want to know what was going on yesterday and only wake up if I put lots of equations on the board or problems that you recognise from the IMO) ... but this means that the course can be followed and understood by mathmos and non-mathmos, and requires only the smallest outlay of mathematical labour to follow. However, the course is *not* intended to be a slight advance of school-level *intellectual* standards, but considerably more challenging and denser. You're at Cambridge now and that means you are amongst the best of the best, and we tend not to do the easy stuff around here. Oh, and don't worry about impostor syndrome: we *all* have *that*.

I presume no particular mathematical background[2] and no technical mathematics will need to be done by the student to follow the course. Any more than historians of medicine are required to carry out dissections and operations, or historians of chemistry are required to go and do alchemical exercises and inhale large quantities of arsenic and mercury vapour. You may feel that these lacunæ are disappointing, and we should make the history of the sciences a little more … *hands on*. Mathmos may find the course of interest as a longer history of what pre-modern mathematics was like, and because it isn't quite the same perspective on the *nature* of the subject that Tripos lectures tend to give.

The lectures are also supposed to be an opportunity to think about *mathematics*, which is a slightly different project from regular maths faculty lectures and a bunch of awesome and mind-blowing theorems you need to be able to prove and apply. It is supposed to be for your amusement (for some fairly strange definition of amusement … but you're mathmos, so that's OK), and it is more about listening and thinking than taking notes and not understanding what the lecturer is talking about. Attendance will require a sense of humour, however, and you should probably think carefully before you show your parents or younger siblings these lectures.

If you are not already signed up to my lectures-email list please do so via the QR below or at www.lists.cam.ac.uk, search for maths-history-lectures. Room changes, reminders, and the such will be announced in the emails.

Piers Bursill-Hall
for the CUHoMSoc
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*[2] Knowing arithmetic might be helpful, but not necessary; brain death, however, would be more of a problem for those considering following these lectures. I will presume the Fermi audience principle (infinite ignorance and infinite intelligence).